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A Framework for the Utilization of Artificial Intelligence in College Mathematics Instruction

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Abstract: *This study provides a framework for utilizing artificial intelligence (AI) in the college mathematics classroom. First, it reviews current trends in mathematics education, as they relate to active learning. Historically, much mathematics instruction has been done in the traditional mode of a non-interactive lecture given by a faculty member, a format where the learner behaves passively while the lecturer delivers information in recent years, more student-focused instructional methods have gained some popularity. The review of the literature provided herein includes an examination of the use of various techniques in college mathematics instruction. We look at instructional techniques that can be used in addition to or instead of purely didactic lecture-based methods. In contrast to the prior format, the lesson examples provided toward the end of this study present approaches that shift the learning paradigm from a model where the teacher is in complete authority to a participatory model where learners and educators together decide how curriculum is delivered and how learning outcomes are assessed by identifying, examining, and selecting modes of delivery and assessment. Following this, we look at topics related to the use of AI in the mathematics classroom. Since the use of AI, especially in the classroom, is a relatively new development, the literature in this area is still in its early stages. Next, this study develops a theoretical framework offering educators the ability to structure lessons on a variety of mathematical topics with both AI and more traditional instructional methods. This study concludes with three sample lessons, with the latter presenting examples of the utilization of the framework at various levels of college mathematics: developmental, core, and upper-level math major courses. The lessons each include an objective, procedures (including both AI-based and non-AI based instructional methods), and a listing of the knowledge, skills, and values acquired in the lesson.*

Keywords*: Artificial intelligence, college mathematics instruction, active learning, andragogy*

1. Introduction

College level mathematics instruction can pose challenges for both students and faculty. Anecdotally, informal conversations with students suggest that "being talked at for an hour" is no longer an effective instructional strategy (if it ever was in the first place). Hence, this study aims to develop a framework for creating college mathematics lessons incorporating active learning and artificial intelligence (AI). The intent is to implement a multifaceted instructional approach that will engage modern learners and appeal to their multiple intelligences. The framework we develop is grounded in extant pedagogical literature and best practices, and the associated literature review appears in the following section.

Following the literature review is the study's theoretical framework designed to facilitate the delivery of college math instruction utilizing active learning and artificial intelligence. Note, however, that more traditional modes of instruction are not completely dismissed. Rather, the framework integrates modern technology into instruction that gives students classroom experiences both logically formatted and eclectic in style.

Finally, three sample lessons illustrate the framework. These lessons run the gamut of topics from developmental mathematics to upper-level mathematics major classes.

2. Review Of Literature

This review of literature aims to provide a framework for the consideration of effective practice in undergraduate higher education mathematics instruction. The framework establishes a structure for supporting examples of mathematics instruction that take conventional pedagogical strategies and adapt them to applications of artificial intelligence. Accordingly, the literature cited describes pre-AI approaches to teaching and learning that have the potential for incorporating AI into those approaches.

Gregory et al (2014) note that calculus students who are fully engaged in the learning process, especially when group activities are invoked, achieved at levels higher than students who essentially worked alone. Students who participated in Facebook groups tasked with calculus assignments delivered higher scores in tests and homework than did those who completed identical assignments individually (Gregory, P., et al, 2014). Nevertheless, the popular thinking seems to suggest that lectures dominate the instructional terrain in mathematics education. Johnson et al attempt to refute that notion. Without analyzing learner outcomes, the authors still found that alternative approaches to traditional lecture, indeed, prevailed in the preponderance of abstract algebra courses included in their study (Johnson, E., 2019). Consequently, the authors concluded that the study's instructor's desire to vary the lecture method by including learner-to-learning activities promoted perceived engagement.

A comprehensive study conducted by Keynes and Olson illustrates the enthusiasm held by senior faculty to mix up instructional techniques (2002). The authors identify widespread support for pedagogical variety, while acknowledging a growing influence of technology in teaching and learning. Findings from the study suggest that changes in pedagogy are overdue and should emphasize cooperative learning paired with technological applications. Laursen and Rasmussen (2019) support the recommendations advanced by Keynes and Olson. Using an inquiry approach-based foundation for instruction, Laursen and Rasmussen (2019) structure their approach, Inquiry-Based Mathematics Instruction, upon four pillars: meaningfulness of the academic discipline, learner collaboration, direct instructor involvement, and fair and inclusive learner engagement. Predating the work of Laursen and Rasmussen is the establishment of a theoretical foundation for teaching. Rasmussen and Kwon (2007) discuss the significance of a healthy classroom climate which is fostered by the implementation of inquiry approaches to the instruction of and the academic achievement in differential equations. The authors conclude that a shift from lecture to inquiry enhances learner empowerment, ownership of the subject-matter, and successful outcomes. Indeed, Artigue and Blomhohj (2013) contend that curricular models allowing time for intensive projects and interdisciplinary study produce classroom environments well suited to inquiry-based learning and the benefits it brings.

Flowing from the preceding observations and recommendations, research conducted by Van et al (2006) and by Andrews et al (2022) shift the perspective from pedagogical considerations to psycho-social perspectives. Millis (2023) contextualizes the perspective of Andrews et al by providing extensive evidentiary findings and theoretical values of infusing cooperative learning into undergraduate courses, including but not limited to developmental mathematics. Differentiating instruction comports with the promise of Millis' observations through the assertion that it levels the playing field for learners with diverse needs (Chamberlin and Powers, 2010). Mogelvang and Nylehn (2022) get more specific in their review of cooperative learning in undergraduate mathematics. Their rationale and subsequent conclusions focus on the unpredictability of the future and the sustainability of cooperative learning in the advancement of academic success. These observations and conclusions, however, would go unfounded in twenty-first century higher education pedagogy without recognizing the dynamics of the teaching and learning environment in culturally diverse classrooms. Croom (1997) offsets the discouraging conclusions of Leyva et al (2021), who suggest that failure to attend to the significance of cultural and ethnic diversity generates classroom dysfunctionality. Croom lays a foundation for addressing diversity and equity, thereby facilitating the development of an egalitarian teacher/student classroom environment and consequently diminishing the prospects of dysfunctionality. Accordingly, Gay (2018) proposes in her seminal volume that deliberately and intentionally recognizing and honoring varied cultural systems and how those systems operate in the classroom enhances a sense of learner self-efficacy and promotes academic achievement.

Van et al (2022) move the needle closer to a potential implementation of AI by experimenting with the efficacy of web-based instruction on improvements in learner self-perceptions (2006). Their article foreshadows beneficial applications of AI when it reveals marked reductions in anxiety and enhancements in self-esteem among subjects taking undergraduate statistics. Although not precisely AI, web-based instruction as a non-human replacement for in-person group interaction had a positive impact on learner self-efficacy. A contemporary view of the instructional potentials resting within non- human interventions emerges from the findings of Andrews et al (2022). Their study opens the door through which conventional pedagogy may cross a threshold leading to the incorporation of non- human assistance in facilitating evidence-based teaching. In crossing that threshold, a Harvard University blog with Chris Dede (Harvard Graduate School of Education, 2023) presents him opining on the value of AI as a naïve interlocutor. In Dede's opinion, AI can ask probing – yet uninformed – questions of the learner, who consequently must formulate informed responses to AI's questions by invoking a combination of prior human knowledge and self-motivated inquiry.

Professional, personal, and cultural circumstances all play roles in the application of varied approaches to undergraduate mathematics instruction and the learning process. Notably, these circumstances lead to the conviction that lecture alone does not facilitate learning as effectively as lecture does – combined with learner-to-learner interaction – enhance the process. For this review, Howard Gardner (2023) fittingly asserts, "We need to understand our human nature – biologically, psychologically, culturally, historically, and pre-historically. It's … the optimal way to launch joint human-computational ventures … "

Let us now examine how and where AI can fit into the process. The following sections portray the transition from what is to what is possible.

3. Toward a Model for the use of AI in undergraduate math instruction

This study establishes a framework for utilizing AI in undergraduate mathematics instruction. Based on the literature review the key pieces in developing an effective delivery method for undergraduate mathematics courses include incorporating the following four components into mathematics lectures and instruction: student engagement in the learning process, integration of technology, inquiry-based learning, and differentiated instruction. The framework does not eliminate lecture; rather, these additional instructional components complement it. AI fits neatly into promoting the implementation of these components in classroom instruction.

Throughout history, reformers of mathematics have argued that the way math was being taught was no longer effective (Phillips, 2016). At the center of this dispute was concern that the current mathematics curricula and teaching methodologies were not providing students with the skills and competencies required to be successful in daily life, at work, and in society at large. In response to Russia's successful launch of the Sputnik in 1957, the United States began what has often been described as a mathematics revolution (Rappaport, 1976). Fear that the Soviet Union would surpass the US in technology caused America to reevaluate how math was being taught in schools. Consequently, it was the desire to keep up with, and even dominate, technology that was the catalyst behind the new math movement (1960-1970). New math was intended to provide a deeper understanding of mathematical concepts by emphasizing reasoning, critical thinking and problem-solving over rote memorization and calculations. It was the idea that students could look at math through the lens of real-world mathematical challenges rather than traditional textbook arithmetic problems. However, the new math movement faced considerable backlash and by 1970, a new movement, back-to-basics, was embraced (Berry, 2005). As outlined by Ellis and Berry in The Paradigm Shift in Mathematics Education: Explanations and Implications of Reforming Conceptions of Teaching and Learning, mathematics reforms introduced throughout the 20th century were continually being challenged (Berry, 2005). Eventually in what the authors coined a cognitive-cultural paradigm (CCP), the emphasis eventually "… shifted from seeing mathematics as apart *from* the human experience to mathematics as a part *of* human experience and interaction" (Berry, 2005). The goal was to get students interested in mathematics rather than making students memorize abstract concepts.

The drive to transvaluate traditional mathematic theorems and methodologies is not a modern phenomenon. Advancements in mathematics have also been greatly motivated by necessity. Geometry was introduced by the ancient Egyptians who needed a technique for measuring land masses changed by the annual flooding of the Nile River for taxation purposes (Burton, 2007). Scientist and mathematician Issac Newton invented[1] the concept of calculus in search of a way to describe his laws of motion and universal gravitation (Roy, 2021). More recently, the development and widespread use of computers has contributed to the development of chaos theory, cryptographic algorithms, and fractal geometry. In his work, *Innovation in Mathematics*, Paul B. Halmos noted that "Mathematics is improving, changing, and growing every day (Halmos, 1958). Halmos further explains how these changes are not solely the result of intellectual inquiry, but also spurred on by the "…bread-and-butter circumstances of our daily lives." (Halmos, 1958)

[1] While there has been much debate over who should be credited with discovering calculus, Isaac Newton, or Fermat Leibniz, Newton's contribution is pointed out to demonstrate how mathematics was used to find a solution to a problem. The authors acknowledge that many thinkers and civilizations contributed to the evolution of calculus.

If instructors are going to introduce mathematical concepts by tapping into the circumstances of their students' daily lives, then utilizing the most recent technologies is critical. Students are entering colleges and universities with more computing skills than previous generations. This digital shift can be attributed to rapid advancements in, and availability of, technology. Today, younger generations. often referred to as digital natives[2] or generation alpha[3], are growing up in environments saturated with digital devices. In contrast, some educators may not have had the same exposure or experiences with evolving technologies, creating a gap in technological skill gap between them and their students. This disparity can sometimes present challenges in educational settings, but it also provides opportunities for collaborative learning and mutual exchange of knowledge. One way to bridge this gap is by utilizing an andragogical teaching approach. With an andragogy teaching model, students are motivated to learn by internal factors, such as the desire for personal development and self-improvement. By removing traditional instructor-pupil constructs, the relationship between the educator and the student becomes more of a partnership. These changes are not being made to merely suggest tweaking mathematical curricula, but out of necessity. Failure to change will inevitably lead to math becoming an abstract subject for generation alpha who may quickly begin to lose interest.

A failure to excel in mathematics as technology evolves and becomes more accessible has been gradually taking place since the early 1980s. The inverse correlation between the increasing number of households that report owning at least one computer[4] and the decline in student math scores cannot be dismissed as mere coincidence.

The number of American households with at least one computer rose from 8.2% in 1984 to 77% in 2010 (US Census, 2022). Today, 93.1% of all US households own at least one computing device (US Census, 2023). However, as the number of households with computers and Internet access increased, the national math skills average for students entering college dropped from 19.8 out of 36 in 2022, to 19.5 in 2023 (ACT, 2023). It is relevant to note that according to ACT (American College Testing), 2022 was the first time in 30 years that the math scores fell below the 20-point threshold (ACT, 2023). While data demonstrates that COVID had a significant impact on college preparedness, educators cannot dismiss the fact that the decline in math scores predates the pandemic (OECD, 2022). Other variables relevant to this decline include the redesign of the college entrance exam 2016, evolving pedagogical paradigms, changing testing environments, and socioeconomics. While the inverse correlation between home computer usage and decline in math scores is not necessarily causal, it does add one more variable to the list of potential reasons behind the decrease in mathematics proficiency.

Introducing new instructional methodologies or technologies into the classroom may not necessarily be met with enthusiasm. Technologies which have been adopted without resistance tend to be those technologies that support traditional teaching such as smartboards, projectors, visualization software, digital libraries, and more recently, learning management systems (Munro, 2008). Factors that contribute to a hesitancy to change include a strong commitment to existing pedagogical traditions, skepticism, lack of experience or familiarity with emerging technologies, and the impact these new technologies might have on student learning (Lucas, 2000). Additionally, as universities strategize how to survive low enrollment numbers by tightening the belt and restructuring departments and programs, faculty are being

[2] Popularized by Marc Prensky in his 2001 article titled "Digital Natives, Digital Immigrants", digital natives refer to individuals who grew up in the digital age, surrounded by technology from an early age.
[3] The term Generation Alpha was first coined by Australian social researcher Mark McCrindle to describe those born from 2010 onward. Members of Generation Alpha are fully immersed in the digital world from birth, growing up in an era of rapid technological advancement.
[4] Computing includes desktops, laptops, tablets, and smart phones.

burdened with increased responsibilities. These extra tasks leave educators with limited time to learn, develop lessons, and implement new technologies into their curriculum. Compounding this situation further is the rapid speed at which AI technologies are advancing.

Research has shown that the use of calculators in mathematics facilitates student motivation and proficiency (Robert Boyle, 2015) (Richard J. Daker, 2021). Initially met with resistance, albeit still a topic of debate (Kakaes, 2012) (Orzel, 2010), the use of calculators in college mathematics is now widely recognized as a valuable instructional tool. Learning mathematics with the aid of a calculator can foster a deeper understanding of the subject and how it relates to the real world, which helps the student appreciate the bigger picture. In "Thinking (and Talking) About Technology in Math Classrooms", Paul Goldenberg writes that "…allowing technology to perform a computation freed students not to think about the computation so that they could focus attention on some other aspect of the problem" (Goldenberg, 2000). Although the author was referencing K-12 learners, this concept applies to college students as well. Basically, instead of focusing primarily on the *how*, technology also enables the student to understand the *why*, which lays the foundation for intellectual inquiry.

Anxiety is another hurdle math teachers are continually trying to avert in the classroom (Eihab Khasawneh, 2021) (Kristina Higgins, 2017). Ironically, some of the pedagogy practices teachers employ may inadvertently be contributing to math anxiety. These practices include requiring a singular, rigid approach for solving an equation or covering problems in a math textbook sequentially (Joseph M. Furner, 2022). Artificial intelligence can aid in reducing math anxiety by providing the student with a personalized learning platform where they can learn at their own pace. AI algorithms can adapt to student emotions through computer vision[5], Natural Language Processing (NLP)[6], and behavior changes (i.e. typing speed, mouse movements, hesitation).

Quawaqneh and Alawameh (2023) tackle issues of mathematics anxiety and its impact on motivation in a study wherein students participated in virtual math labs based on AI. Upon completion of the study, learners who participated in this mode of delivery motivationally surpassed their counterparts who participated in either virtual laboratories alone or in conventional classroom teacher/learner instruction. The authors concluded that the AI/virtual laboratory group demonstrated stronger measures of motivation toward learning mathematics than did their peers who engaged in the other two instructional formats. Further, the plain and simple notion that the job of learning mathematics is an onerous (reluctantly approached at best) undertaking gives meaning to the work of Kaushik, Parmar, and Jhamb (2021). They assert that AI-supported instruction comports diversity in its manifold presentations: cultural, developmental, and intellectual. Its consequence is a plain and simple one: AI in culturally, developmentally, and intellectually diverse environments promotes a willingness to engage.

Traditionally, teachers provide feedback to students by grading assignments and quizzes or through formative assessment in the classroom. However, unless the instructor is working one-on-one with the student, feedback is delayed. This feedback lag can be reduced with the use of artificial intelligent technologies which provide real-time feedback. The personalized support and resources, (David Williams, 1985)tailored to the student's specific learning needs, promotes increased engagement, fosters self-directed learning, and increases the likelihood that the student will retain the information learned. At a time when universities are struggling financially due to low enrollment, many institutions are increasing

[5] Algorithms that enable machines to gain a high-level understanding of images or videos the way humans interpret visual data.
[6] NLP involves algorithms and computational models that enable machines to understand, interpret, and generate human-like language.

class size to maximize financial efficiency. Unfortunately, with more students in the class, instructors have reduced opportunities for individualized interactions. While there has been long debate over the impact of classroom size on learning (David Williams, 1985) (Linda Toth, 2002), research has shown that students from socioeconomically disadvantaged populations do not fare well in larger classes (Bressoux, 2005).

It would be rare today to explore AI in academia without discussing ChatGPT conversational chatbot[7]. Developed by OpenAI, ChatGPT is an AI-powered language model which has gained much popularity since its release on November 30, 2022. Reminiscent of the forty-year debate and initial resistance over calculators and computers in the classroom, many educators fear that ChatGPT will encourage cheating and negatively impact learning. According to a 2023 report from Cengage, The Faces of Faculty, 43% of all faculty surveyed responded that AI-generated plagiarism is a top concern compared to 37% in 2022 (Cengage, 2024) Some of these concerns may very well be valid if technologies like ChatGPT are used incorrectly. However, banning or putting restraints on ChatGPT and other NLP usage in the classroom would be doing students a disservice because these technologies are already being implemented in industry. The focus should be on ethics, privacy, responsibility, and how to use generative language models correctly (i.e. prompt engineering). It is also critical for students to learn the limitations of programs like ChatGPT.[8]

AI can enhance critical thinking in mathematics by helping them to become independent learners. According to Kopzhassarovaa et al. (2016), students are most motivated to learn when the content being taught is meaningful and of interest to them. Artificial intelligence can provide real-word scenarios that are relevant to the student. This helps the student see math as tangible rather than some abstract concept they will never use in their daily lives. Through repeated use, the AI system can collect data about the student's performance, engagement metrics, and learning preferences. Analysis of this data will help the program identify the student's preferred learning style and adapt accordingly. These individualized data analyses can be extremely valuable to educators for assessment purposes.

It is significant to point out that ChatGPT is not the only AI-driven program available. Other tools available include DALL-E (text-to-image generator), Teachable Machine (creates machine learning models), POE (access multiple chatbots from one hub), Gemini (formerly Google Bard), SymbMath (symbolic mathematics tool), and MyScript (simplifies challenging math problems).[9]

Traditionally, educators could attend workshops or faculty training sessions to learn about emerging technologies before implementing them in the classroom. However, artificial intelligence is advancing so rapidly that it leaves little time for reskilling or upskilling. This presents a challenge for educators who may be unfamiliar with emerging AI technologies. Another dilemma is that oftentimes the decision to use, or not use, generative models like ChatGPT in academia are being made by administrators and individuals who have little or no technological background.

In his seminal book of essays, *From Digital Natives to Digital Wisdom*, (Prensky, 2012) Marc Prensky describes individuals born after 1980 as digital natives because they grew up immersed in technology. According to Prensky, from a young age digital natives have had access to computers, cell phones, video games, digital tablets, music players and toyboxes full of smart toys (Prensky, 2012). When visiting an

[7] The acronym GPT stands for Generative Pre-trained Transformer.
[8] Limited knowledge, data not the most current, character limitations, inability to multitask (Narrow AI), potential bias, and errors.
[9] This is not an extensive list, just a few examples.

airport or restaurant, it is commonplace to see toddlers being kept occupied with their parents' cell phones. Even baby bouncers, rocking chairs, and highchairs now come with iPad holders. These youngsters are learning how to swipe and process information before they can talk or walk and will eventually even surpass Prensky's digital natives.

In *Grown Up Digital*, Researcher Don Tapscott describes how growing up saturated with technology has changed the next generation (next gen) (Tapscott, 2009). For the next gen, who were "bathed in bits" (Tapscott, 2009), technology is not extrinsic, but rather an intrinsic part of who they are. Based on over 10,000 interviews, Tapscott and his research team concluded that "…education should not focus on transmitting knowledge, but on teaching students how to learn" (Tapscott, 2009). For Tapscott, "Teachers should shift from lectures to interactive, collaborative guidance, and let students explore and discover on their own" (Tapscott, 2009). Thus, an andrological or hybrid teaching approach is more suited to Tapscott's next gen students and forthcoming generations.

Anthony S. Bryk, president of the Carnegie Foundation for the Advancement of Teaching (CFAT), referred to developmental math classes as the "graveyard where dreams go to die" (Kimberley Gomez, 2015). According to the American Heritage Dictionary, the word graveyard means a place where "worn-out or obsolete objects are kept" (Houghton Mifflin Company, 2015). However, it is not the dreams per se, but rather outdated instructional techniques which are worn-out. Many of the students who score low on math placement tests are (1) first-generation college students (2) students coming from lower socioeconomic backgrounds or (3) students who received little to no math education prior to entering college (APA, 2024). The challenge then becomes how to teach mathematics to students who view math as intangible. Perhaps a good place to start is by removing the phrase "developmental math" and replacing it with "exploring math". Although developmental supplanted the even more archaic remedial math, both words can have negative connotations.

4. Lessons

We now propose several sample lessons as practical examples of how AI can be used in the mathematics classroom. Each sample lesson has the following format: (1) Objective, (2) Procedure, (3) non-AI activity, (4) AI related activity.

Part 1 Exploring Fractions:
Objective: Students will conceptualize fractions, add and subtract fractions with like denominators, and simplify answers.

Procedure:
1. Learners will explain their prior knowledge of fractions by responding to the following inquiry:
- What is a fraction?
- Can you share examples of fractions in real life?
- In your own words, how would you describe the numerator and denominator of a fraction?
- What does the numerator tell us?
- What does the denominator tell us?

2. Learners will review the basics of fractions placing an emphasis on the function of a fraction representing a part of a whole and consisting of a numerator (top number) and a denominator (bottom number). Pairs of students will use manipulatives to complete four to six basic fraction problems. The

instructor will provide a brief history of fractions (Egyptians, Babylonians, Chinese, Fibonacci, etc.). Familiarization with the history of mathematics also invigorates diversity, equity, and inclusion (DEI) initiatives by fostering representation, cultural understanding, and the empowerment of marginalized students.

Sample Activity: Dividing a circle in halves/quarters/eights.
Principle question (the anticipatory set): Using manipulatives, how would you divide a circle in halves/quarters/eights?

Follow-up activities/questions:
1. How do you know your answers are correct?
2. Can you support your response by demonstrating and describing the process leading you to these conclusions?
3. Are there optional approaches?

Additional problems tied to the three follow-up questions ensue. Problems grow increasingly more complex, and reiteration of follow-up questions enables students to strengthen their understanding of fractions:
First, divide the circle in one third/sixth/twelfth, three quarters, sixth eights, continuing in increasing complexity.
Second, replicate the manipulative-based problem-solving activity with solutions incorporating mathematical notations. In a guided discussion introduce the concept of like fractions where students acknowledge that fractions with the same denominators can be readily added and subtracted. Evoke this understanding by asking:
- Which methodology was easier?
- Why is mathematical notation necessary?
- Are there other ways to do this?

Third, task the pairs with designing) an Artificial Intelligence program (or game) that could solve complex fractions more efficiently. Encourage creativity by asking:
- What would an ideal program look like?
- How would it function?
- How would it help humans?
- Would this program replace mathematicians? Justify.

Direct students to conclude that mathematics is an evolving discipline, constantly shaping and introducing methods, theories, and approaches (i.e.: from the abacus to the calculator).

Acquired knowledge, skills, and values:
- Knowledge: conceptualization of fractions
- Skills: recognition of a fraction; definition of a fraction
- Values: applicability of fractions; collaboration

Part 2 Exploring Fractions with AI:
Example 1

Objective: Use of AI-powered educational tools will reinforce conceptualization of fractions from practicing addition and subtraction with like denominators and receiving immediate personalized feedback on progress.

Procedure:

1. Students will use AI-powered educational tools:
   - Students search the Internet and locate an open-source AI-powered educational platform with interactive exercises enabling students to communicate directly with the AI tool (instructor can provide a tool if they prefer).

2. Students will reinforce conceptualizations of fractions:
   - Students will visually manipulate fraction pieces like the fraction manipulatives used in Part 1 (for example, instead of drawling circles, students might drag and drop fraction circles to add or subtract fractions).

3. Students will receive personalized feedback:
   - The AI tool will provide immediate feedback to students as they work through the exercises. If a student makes a mistake, the AI program can provide hints or explanations to guide them toward the correct solution. Conversely, an incorrect response from AI can flip the roles putting the student in the role of teacher training the AI model, thus strengthening their critical thinking skills.

4. Students will engage in discussing and summarizing their experiences with AI-powered educational tools:
   - Students will share impressions about the tool.
   - Students will informally assess the quality of their understanding of fractions.
   - Students will gauge their overall experience and comprehension of fractions after using the AI tool.

Sample Activity: Applying AI in a core level undergraduate math course
Principle question, the anticipatory set: How will using AI enable you to calculate and interpret probabilities within the context of normal distributions?

Follow-up activities/questions:

1. Analyze the theory behind what normal distributions are.
2. Delineate the many applications to which they are suited.
3. Discuss, examine, and provide examples related to two essential questions:
   - How can you use a standard normal table
   - How do you standardize a value using the formula $Z = \frac{x-\mu}{\sigma}$, where μ and σ are the mean and standard deviation, respectively, of a normal distribution.

Once students' comfort levels are reached with the basic theory and calculations, the students can delve deeper into the basic theory and calculations by engaging in small group problem-solving sessions:
First, students select problems most suited to their interests (e.g. applications in sports analytics, biology, etc.). An example of one such problem provides students with the mean and standard deviation of the weights of two breeds of dogs, assuming the weights follow two distinct normal distributions.
Second, Student groups receive weight data of one large dog for each of breed and apply conceptualizations acquired in the introductory section of this module to determine:

(a) the weight percentile for each of these two dogs (within their respective breed), and
(b) based on these percentiles, which dog is more unusually large.

Use of AI

1. Students search the Internet to locate an open-source AI-powered educational platform (instructor provides a tool if they prefer).
2. Employing the results of the normal distribution exercises delineated previously, students will use their chosen (or provided) AI tool to calculate the same probabilities as in the weight percentile example.
3. Students will compare student-generated answers to AI answers for the purpose of identifying and resolving discrepancies, i.e., was either or AI wrong?

Students will inquire into the question, "which dog is more unusually large?" for the purpose of participating in small group discussion about the effectiveness and accuracy of the AI tool.
Acquired knowledge, skills, and values:

- Knowledge: Basic normal distribution theory and calculation
- Skill: Selection of appropriate AI-powered educational platforms
- Value: Recognition that that AI is one more tool in the box

Example 2
Objectives:
1. In an upper-level course for mathematics majors, students will prove that in hyperbolic geometry, given a line and a point not on that line, there exist infinitely many lines through that point parallel to the given line.
2. Students will recognize errors in mathematical theorems.
Procedure:

1. Students will prepare written proofs for a mathematical theorem, meeting in triads to assess each other's work.
2. Students will discover that each proof may be correct noting that not each proof may be identical to the other.
3. Triads will collaborate on correcting proofs that result in incorrect findings. Upon reaching consensus that proofs are correct, triads will submit them for examination by the instructor.
4. The instructor will examine each proof and provide feedback on the quality of the proof.

Sample activity: AI-powered resources in an upper-level math course
Principle question, the anticipatory set: How will AI proofs differ from student-generated proofs, and how can student proofs and AI proofs each yield correct results?

Follow-up activities/questions:

1. Search the Internet and find an open-source AI-powered educational platform (instructor can provide a tool if they prefer).
2. Students will ask the AI tool to prove the theorem.
3. Students will analyze the AI tool's proof and apply the design of their previously developed proof to the design of the AI proof, looking for the presence of any errors in their analyses.
4. If errors surface in the AI generated proof, students will teach the AI tool to prove it correctly by:
   - informing the AI tool of individual errors in its proof.
   - advising the AI tool of faulty relations drawn between individual pieces of the proof.

5. Students will instruct the AI tool to generate a correct proof and will compare the AI proof to the original student-generated proof.
6. Students will cite any differences found between proof writing techniques.

Acquired knowledge, skills, and values:
- Knowledge: Theorem proof production
- Skills: Selection of appropriate AI tools; teaching AI resources
- Value: utility of AI tools in advanced mathematical operations

As a specific example of attempting the proof above using ChatGPT, please see figure 1 –insert screenshot- In this case, students might note that the proof given by the AI tool in this example is circular. They might then try to prompt ChatGPT to improve its proof by telling it to “use the crossbar theorem.” This is just one example, of many ways, in which this module could play out using AI. There are infinitely many ways this student-AI interaction could evolve. details of this evolution depend on the AI platform used, the initial quality of the AI generated proof, and the prompts the student gives to the AI platform.

Applications of AI in upper-level math courses require elaborating upon the knowledge, skills, and value delineated above. In the case drawn from hyperbolic geometry, where the activity incorporates AI into proof activities, it is important that students recognize the circular nature of the AI tool’s proof. Upon recognizing AI’s weakness, students should be encouraged to “teach” AI to improve its proof by instructing it to “use the crossbar theorem,” an example representing just one illustration of a multitude of potential student-AI interactions where students initiate the interaction. This kind of interaction is posited upon three conditions: the AI platform used, the original quality of the AI generated proof, and the prompts given by students to the AI platform.

5. Conclusion

In conclusion, this work presents a coherent framework for developing contextual mathematics instructional lessons that includes the use of artificial intelligence. This framework is thoroughly based in the literature and is certainly not *limited* to the use of AI. Rather, AI is used as yet another tool to help students actively learn material in the mathematics classroom (and outside of the classroom). Three sample lessons were also provided which provide examples of how this framework can be used to develop lessons at any level of college mathematics.

Given the dynamic nature of AI, the potential for future research in this field is limitless. Of course, the technologies of tomorrow are not known with any certainty, so it is not possible to say what mathematics instruction might look like 10 years from now. That said, the logical framework provided in this paper can be adapted for use to whatever new technologies and resources are available to students in the future.

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